19930092719

ARR No. 3K15

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

WARTIME REPORT

ORIGINALLY ISSUED

November 1943 as

Advance R[redacted] Report 3K15

A COMPARATIVE STUDY OF THE EFFECT OF WING FLUTTER SHAPE ON THE CRITICAL FLUTTER SPEED

By I. Irving Pinkel

Langley Memorial Aeronautical Laboratory
Langley Field, Va.

WASHINGTON

L - 413

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

ADVANCE [redacted] REPORT

L-415

A COMPARATIVE STUDY OF THE EFFECT OF WING FLUTTER SHAPE ON THE CRITICAL FLUTTER SPEED

By I. Irving Pinkel

SUMMARY

A comparison is made of the results of calculations of the critical flutter speeds of nine uniform rectangular wings without ailerons by two methods, as follows:

1. The method in which the effect of the wing flutter shape is introduced indirectly by choosing mean values for the wing parameters

2. A method that introduces an additional degree of freedom in the wing-bending flutter mode

These examples show that, for the case of normal wings and even for wings of higher aspect ratio, the difference in the values obtained with these two methods is inconsequential and the effect of including a second degree of freedom in the bending mode is usually small.

INTRODUCTION

The effect of the shape of a fluttering airfoil is introduced into the equations of motion and a comparison is made of the predicted critical flutter speeds obtained by the several methods that have been developed for solving the flutter problem.

Lagrange's equations, applied to the flutter problem, provide a means of solving for the critical flutter speed and frequency and, in principle, for the wing flutter shape. In the present paper, flutter speeds predicted by the use of this method for uniform wings of rectangular plan form are compared with the results obtained by the standard two-dimensional method (reference 1) in which the effect of wing flutter shape is only indirectly introduced. Both values are compared wi[redacted] speeds obtained from experiments made in the 8-foot high-speed tunnel at LMAL. This comparison is used to judge the importance of introducing considerations of wing flutter shape into the flutter problem.

A solution of the flutter problem was given by Theodorsen (reference 1) and its application and implications were amplified by Theodorsen and Garrick in a subsequent paper (reference 2). The formal solution of the flutter problem was carried out on a uniform infinite wing about which the air flow is two-dimensional. The structural stiffness of the wing in bending and torsion is represented by equivalent springs. The aileron is assumed to extend the entire length of the wing and is supported at its hinge by a spring representing the elastic properties of the aileron-control system. The wing has three distinct types of vibratory motion: bending motion perpendicular to the plane of the wing, torsion motion about the static torsion axis, and aileron motion about the aileron hinge. Flutter is a sustained oscillation involving two or more coupled vibration modes. Four types of flutter are thus recognized: (1) bending-torsion; (2) bending-aileron; (3) torsion-aileron; and (4) bending-torsion-aileron. The primary parameters determining the critical flutter speed are found to be: the wing chord, the mass of the wing per unit span length, the chordwise position of the static torsion axis and center of gravity, the moment of inertia of the wing about the static torsion axis, the air density, the torsion and bending stiffness, the aileron center-of-gravity position with respect to the aileron hinge, the aileron hinge position on the wing chord, the aileron mass per unit length, and the stiffness of the aileron-control system. The mass and stiffness parameters are combined in the equations of motion of the wing to give nondimensional coefficients that define the bending, torsion, and aileron natural vibration frequencies.

APPLICATION OF THE TWO-DIMENSIONAL FLUTTER THEORY TO FINITE WINGS

When the two-dimensional flutter theory is applied to a wing that is a finite continuous structure with section parameters variable along the span, several questions are raised:

L-413

1. What are the air forces assignable to a wing vibrating with amplitude variable along the span?

2. Which section parameters of the wing are to be used in solving for the critical flutter speed?

3. What values are to be given to the frequency terms occurring in the theory?

The present practice in applying the two-dimensional flutter theory is to pick a position along the wing span that is considered representative of the average properties of the wing with regard to section parameters and wing flutter motion. The three-quarter semispan section is normally considered to be this characteristic wing section. The lowest natural vibration frequencies in wing bending, torsion, and aileron oscillation are assigned. The solution for the critical flutter speed is then assumed to be that of a uniform infinite wing having the described properties. The air forces used are those obtained by a two-dimensional wing treatment. Good judgment in selecting the representative wing section gives calculated critical flutter speeds that come close to experimental values.

When one or both of the coupled modes of vibration making up the flutter vary in magnitude along the span or do not involve the entire wing, the effect of the wing shape in flutter is introduced into the problem by applying weighting factors to the coupling terms present in the equation of motion of the fluttering wing. For two-dimensional-flutter cases of an infinite wing involving the aileron, the weighting factor to be applied to the aileron-bending and aileron-torsion coupling terms would be the ratio of aileron span to wing span. For an actual wing in the case of bending-torsion flutter, the weighting factor ξ would depend on the wing bending and wing torsion shapes in the flutter vibration.

If the displacement of any point of the wing due to the bending mode of vibration is given by $h(x)e^{i(\omega t+\varphi_1)}$ and the displacement due to torsional vibration is written as $\alpha(x)e^{i(\omega t+\psi_1)}$

where

x span coordinate measured from wing root

$h(x)$ amplitude of bending vibration at x

$\alpha(x)$ amplitude of torsional vibration at x

ω flutter frequency

t time

φ_1, ψ_1 phase angles of the respective vibrations

Then, for the case of a rectangular wing with section parameters constant along the span, the coefficient of coupling is obtained by the integral $\xi = \int_0^l h(x)\, \alpha(x)\, dx$ where l is the wing semispan length.

THE USE OF LAGRANGE'S EQUATIONS IN THE FLUTTER PROBLEM

The application of Lagrange's equations and generalized coordinates to the flutter problem provides a means of treating the wing as a continuous structure capable of vibrating in its various modes with amplitude variable along the span. The characteristic determinant developed in the course of the solution of a flutter problem yields upon the solution of the equations derived therefrom an estimate of the shape assumed by the fluttering wing as well as the critical flutter speed and vibration frequency. One weakness of this method at present is the practical necessity for assigning to each wing section the air forces that would exist on a two-dimensional wing vibrating in a manner identical with the section. This assumption leaves out the effect of aspect ratio. For the higher reduced flutter frequencies the error introduced by neglecting the effect of aspect ratio is not large.

In applying Lagrange's equations it is considered that the function which gives the flutter vibration shape along the span for a given case can be made up of a sum of elemental span functions, each multiplied by a factor. These multiplying factors are taken as the generalized coordinates of the problem and their evaluation gives the function expressing the variation of vibration amplitude along the wing span for each vibration mode. If $H(x)$ is the amplitude span function of the bending vibration and $\bar{\alpha}(x)$

the corresponding function for the torsion vibration, then

$$H(x) = \Sigma q_n e^{i(\omega t+\varphi_n)} h_n(x)$$

and

$$\overline{\alpha}(x) = \Sigma p_n e^{i(\omega t+\psi_n)} \alpha_n(x)$$

where

q_n, p_n generalized coordinates

$h_n(x)$, $\alpha_n(x)$ elemental amplitude functions for bending and torsion vibrations

φ_n, ψ_n phase angles of bending and torsion vibration modes

If the wing bending shape in flutter is assumed to be some combination of the wing shapes in the first and second natural vibration modes of the wing, the use of two elemental functions $h_1(x)$ and $h_2(x)$, representing the first and second natural vibration bending modes, is sufficient for the series expressing the flutter bending shape $H(x)$. Because the lowest natural torsion frequency of the wing is so many times higher than the bending frequency, the first natural torsion mode of the wing is assumed to be a good approximation to $\overline{\alpha}(x)$, the torsion flutter shape. The bending and torsion flutter amplitude functions employed are

$$H(x) = q_1 e^{i(\omega t+\varphi_1)} h_1(x) + q_2 e^{i(\omega t+\varphi_2)} h_2(x)$$

$$\overline{\alpha}(x) = p_1 e^{i(\omega t+\psi_1)} \alpha_1(x)$$

For the case of a rectangular wing with constant section parameters, the various bending and torsion natural vibration modes are well known and can be approximated by simple polynomials in x. There is then for $h_1(x)$, $h_2(x)$, and $\alpha_1(x)$

$$h_1(x) = \left(2\,\bar{x}^2 - \frac{4}{3}\,\bar{x}^3 + \frac{1}{3}\,\bar{x}^4\right) N_1$$

$$h_2(x) = (1.6276\,\bar{x}^2 - 4.1621\,\bar{x}^3 + 5.3484\,\bar{x}^4 - 0.9231\,\bar{x}^5) N_2$$

$$\alpha_1(x) = (2\,\bar{x} - \bar{x}^2)\,N_3$$

where $\bar{x} = \frac{x}{l}$, l being the wing semispan length, and N with subscripts represents normalizing factors so chosen that $\int_0^l \frac{h_n^2}{b^2}(x)dx = 1 = \int_0^l \alpha_n^2\,(x)dx$. The values of N thus determined are

$$N_1 = 1.97b$$

$$N_2 = 19.3b$$

$$N_3 = 1.37$$

where b is the wing semichord length. The shapes of these functions are illustrated in figure 1.

For the bending-torsion flutter case for the uniform rectangular wing, the application of Lagrange's equations gives for the generalized equations of motion:

$$\left[A_{a\alpha}\int_0^l \alpha_1^2(x)dx + \frac{1}{\kappa\omega^2}\,\frac{GJ}{Mb^2}\int_0^l \left(\frac{\partial\alpha_1(x)}{\partial x}\right)^2 dx\right] p_1$$

$$+\left[\frac{A_{ah}}{b}\int_0^l h_1(x)\;\alpha_1(x)dx\right] q_1 + \left[\frac{A_{ah}}{b}\int_0^l h_2(x)\;\alpha_1(x)dx\right] q_2 = 0$$

$$\left[A_{c\alpha}\int_0^l h_1(x)\alpha_1(x)dx\right]p_1$$

$$+\left[\frac{A_{ch}}{b}\int_0^l h_1{}^2(x)dx+\frac{1}{\kappa\omega^2}\frac{EI}{Mb}\int_0^l\left(\frac{\partial^2 h_1(x)}{\partial x^2}\right)^2 dx\right]q_1$$

$$+\left[\frac{A_{ch}}{b}\int_0^l h_1(x)h_2(x)dx+\frac{1}{\kappa\omega^2}\frac{EI}{Mb}\int_0^l \frac{\partial^2 h_1(x)}{\partial x^2}\,\frac{\partial^2 h_2(x)}{\partial x^2}dx\right]q_2 = 0$$

$$\left[A_{c\alpha}\int_0^l h_2(x)\,\alpha_1(x)dx\right]p_1$$

$$+\left[\frac{A_{ch}}{b}\int_0^l h_1(x)\;h_2(x)\;dx+\frac{1}{\kappa\omega^2}\frac{EI}{Mb}\int_0^l \frac{\partial^2 h_1(x)}{\partial x^2}\,\frac{\partial^2 h_2(x)}{\partial x^2}\;dx\right]q_1$$

$$+\left[\frac{A_{ch}}{b}\int_0^l h_2{}^2(x)\;dx+\frac{1}{\kappa\omega^2}\frac{EI}{Mb}\int_0^l\left(\frac{\partial^2 h_2(x)}{\partial x^2}\right)^2 dx\right]q_2 = 0$$

The symbols used are consistent with those employed in references 1 and 2. For reference, the terms are listed as

M wing mass per unit length

b wing semichord

EI wing bending stiffness; product of Young's modulus and moment of inertia of wing cross section

GJ wing torsion stiffness

$A_{a\alpha}$, A_{ah}, etc. complex terms defined in references 1 and 2

Performing the integrations indicated for the terms containing wing structural stiffness terms yields

$$\frac{EI}{Mb}\int_0^l \left(\frac{\partial^2 h_1(x)}{\partial x^2}\right)^2 dx = \omega_{h_1}^{\ 2}\ l\ b$$

$$\frac{EI}{Mb}\int_0^l \left(\frac{\partial^2 h_2(x)}{\partial x^2}\right)^2 dx = \omega_{h_2}^{\ 2}\ l\ b$$

$$\frac{GJ}{Mb^2}\int_0^l \left(\frac{\partial \alpha(x)}{\partial x}\right)^2 dx = \omega_{\alpha}^{\ 2}\ l\ r_{\alpha}^{\ 2}$$

The remaining integrals yield

$$\int_0^l h_1(x)\ h_2(x)\ dx = -0.126\ l\ b^2$$

$$\int_0^l h_1(x)\ \alpha_1(x)\ dx = 0.95\ l\ b$$

$$\int_0^l h_2(x)\ \alpha_1(x)\ dx = 0.172\ l\ b$$

$$\int_0^l \frac{\partial^2 h_1(x)}{\partial x^2}\ \frac{\partial^2 h_2(x)}{\partial x^2}\ dx = \frac{5.498\ b^2}{l^3}$$

where

ω_{h_1} first natural wing bending frequency

ω_{h_2} second natural wing bending frequency

ω_{α_1} first natural wing torsion frequency

v critical flutter speed

ω critical flutter frequency

k reduced flutter frequency $= \frac{\omega b}{v}$

r_α radius of gyration of wing mass about static torsion axis

ρ air density

and

L-413

$$\Omega_1 = \left(\frac{\omega_{h_1}}{\omega_\alpha r_\alpha}\right)^2$$

$$\Omega_2 = \left(\frac{\omega_{h_2}}{\omega_\alpha r_\alpha}\right)^2$$

$$X = \left(\frac{\omega_\alpha r_\alpha b}{vk}\right)^2 \left(\frac{1}{\kappa}\right)$$

$$\kappa = \frac{\pi \rho b^2}{M}$$

The generalized equations of motion become, after some reduction,

$$(A_{a\alpha} + X)p_1 + (0.95\ A_{ah})q_1 + (0.172\ A_{ah})q_2 = 0$$

$$(0.95\ A_{c\alpha})p_1 + (A_{ch} + \Omega_1 X)q_1 + (-0.126\ A_{ch} + 0.01095\Omega_2 X)q_2 = 0$$

$$(0.172\ A_{c\alpha})p_1 + (-0.126\ A_{ch} + 0.4366\Omega_1 X)q_1 + (A_{ch} + \Omega_2 X)q_2 = 0$$

The critical values of X and k are obtained by the simultaneous solution of the real and imaginary equations in X and k resulting from the expansion of the characteristic determinant formed from the coefficients of the generalized coordinates of this set of equations. The flutter speed is then obtained from the definitions of these terms

$$v = \frac{r_\alpha \omega_\alpha b}{\sqrt{\kappa}} \frac{1}{k} \frac{1}{\sqrt{X}}$$

With v and k known, the flutter frequency ω is obtained from the expression defining k.

When the values of the various critical flutter constants are substituted in the equations of motion, the generalized coordinates are evaluated and the approximation to the wing flutter shape is obtained.

COMPARATIVE CALCULATIONS OF CRITICAL FLUTTER SPEEDS

Critical flutter speeds have been determined for nine rectangular wings in the NACA 8-foot high-speed wind tunnel at LMAL. For each of these wings six theoretical critical speed determinations were made:

1. Lagrange's equation method for one bending and one torsion mode

2. Lagrange's equation method for two bending and one torsion mode, with $\int_0^l h_1(x)\, h_2(x)\, dx = -0.126\ l = \xi$

3. Lagrange's equation method for two bending and one torsion mode, with $\int_0^l h_1(x)\, h_2(x)\, dx$ arbitrarily set equal to zero $(\xi = 0)$. The coefficient $-0.126\ l$ comes from the term $\int_0^l h_1(x)\, h_2(x)\, dx$ which would be zero if $h_1(x)$ and $h_2(x)$ were a pair of strictly orthogonal functions.

4. Lagrange's equation method when one torsion mode and two bending modes are used; the second mode is a tip deflection made up of that portion of $h_2(x)$ beyond the node occurring about $0.7l$ along the semispan. These computations are given for plastic wings, which showed a tip-deflection tendency in the wind-tunnel tests.

5. Theodorsen's method when the first natural bending frequency is used

6. Theodorsen's method when the second natural bending frequency is used

The justification for making calculations with ξ equal to zero (group 3) is based on the argument that the small change in the shape of the second bending mode necessary to make it orthogonal with the first bending mode is so small that the other coupling terms involving the second bending mode are not changed appreciably. Previous computations have shown that small changes in these coupling terms do not produce an appreciable effect on the computed critical flutter speed.

The critical-flutter-speed computations with

Theodorsen's method using the second bending frequency were made to show that the experimental critical flutter speed usually lies between the computed values obtained with the first and second wing-bending frequencies. This point is discussed in detail in reference 2.

The values of the wing parameters used in these computations are given in table I. The results of these computations, as well as the experimental critical flutter speeds for the corresponding wings, are given in table II.

From a comparison of the values obtained for the critical flutter speeds computed by Theodorsen's method and Lagrange's equation method, it appears that considerations involving the effect of wing flutter shape in the computation of critical flutter speeds may not be necessary in view of the small and inconsistent differences in the values obtained and the increased amount of work introduced.

Wings 2 to 14A were of large aspect ratio and therefore were suitable for introducing additional bending modes into the shape of the fluttering wing. For these wings Lagrange's equation method yields results that do not represent a consistent improvement in the predicted critical flutter speeds as compared with the values obtained by Theodorsen's method.

For wings 16 to 18, which were of plastic material constructed more like actual wings, the values of the critical flutter speeds obtained with Lagrange's equation method using two bending modes, in general, come further from the experimental values than do Theodorsen's. For these wings, the second bending mode introduced into the problem appears to have no effect on the calculated critical flutter speed. Because these wings are weak in bending near the tip, choosing the second bending mode as a tip deflection tends to bring Lagrange's equation values a little closer to the experimental flutter speeds. The tip-deflection function was taken by using the expression $h_2(x)$ and fixing its value as zero up to the position of the node that occurs about $0.7l$ along the span from the root. This effect emphasizes the point that in limiting Lagrange's equation method to a small number of elemental shapes, a great deal depends on the proper choice of these functions.

When one bending and one torsion mode are used in

Lagrange's equation method, the results are, in general, less accurate than when a second bending mode is introduced. Arbitrarily setting $\int_0^l h_1(x)\, h_2(x)dx$ equal to zero reduces the accuracy of the results.

The application of Lagrange's equations to the flutter problem described in this report is similar to that developed in reference 3 by S. J. Loring. In carrying out the computations described in this section of the report the values substituted for the frequency terms in some of the flutter calculations were obtained from wing-vibration measurements which gave coupled frequencies that may differ from the true mode frequency by as much as 10 percent. In this respect the computations with Lagrange's equations made in this report differ somewhat from the method outlined by Loring in which uncoupled values of the wing frequencies are specified. In comparison with the errors introduced by uncertainties regarding the location of the static torsion axis and particularly by neglecting vibration damping, the error in computed critical flutter speed resulting from the use of coupled vibration frequencies is not large.

CONCLUSIONS

1. When limited by practical considerations to the use of two bending-mode functions, Lagrange's equation method gives predicted critical flutter speeds that are close to the values obtained by application of Theodorsen's flutter theory. There is no indication from the values obtained by either method in this study that one method will consistently give predicted flutter speeds closer to the experimental values than will the other.

2. For wings of normal aspect ratio, the introduction of a second bending mode in Lagrange's equation system appears to have little effect on the critical flutter speeds obtained. Good judgment must be used in selecting the elemental function representing the second bending flutter mode if an appreciable effect is to be made on the calculated critical flutter speed with a second bending mode introduced into the problem.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va.

REFERENCES

1. Theodorsen, Theodore: General Theory of Aerodynamic Instability and the Mechanism of Flutter. Rep. No. 496, NACA, 1935.

2. Theodorsen, Theodore, and Garrick, I. E.: Mechanism of Flutter - A Theoretical and Experimental Investigation of the Flutter Problem. Rep. No. 685, NACA, 1940.

3. Loring, S. J.: General Approach to the Flutter Problem. SAE Jour., vol. 49, no. 2, Aug. 1941, pp. 345-355.

TABLE I.- WING PARAMETERS USED IN FLUTTER COMPUTATIONS

Wing	ω_α (cpm)	$\frac{\omega_{h_1}}{\omega_\alpha}$	$\frac{\omega_{h_2}}{\omega_\alpha}$	b (ft)	κ	a	x_α	r_α^2
2	1085	0.0694	0.425	0.500	1/90	-0.400	0.25	0.3125
6	1150	.120	.670	.5012	0.1170	-.1762	.08	.272
10	1470	.0694	.425	.5025	.01138	-.400	.25	.300
11	1790	.0659	.405	.500	.01052	-.400	.254	.3045
12	1925	.0675	.411	.502	.00972	-.251	.08	.248
13	2350	.0642	.400	.500	.00673	-.400	.2316	.2885
14A	2155	.0659	.399	.5013	.00772	-.398	.234	.2897
16	780	.1192	.833	.500	.0746	-.450	.098	.360
16S	795	.132	.906	.500	.0748	-.450	.098	.360
17	1600	.247	1.50	.500	.0349	-.104	.052	.345
18	1650	.252	1.612	.507	.0796	-.450	.098	.360

TABLE II

COMPARISON OF CALCULATED CRITICAL FLUTTER SPEEDS WITH EXPERIMENTAL VALUES

Wing	q_1/p			q_2/p			Calculated critical flutter speeds						Experimental flutter speeds[b]
							Loring's method[a]				Theodorsen's method[a]		
							One bending mode	Two bending modes			First bending-mode frequency	Second bending-mode frequency	
	$\xi=-0.126$	$\xi=0$	Tip deflection	$\xi=-0.126$	$\xi=0$	Tip deflection		$\xi=-0.126$	$\xi=0$	Tip deflection			
2	1.762	2.018	-----	1.221	0.990	-----	232	212	221	-----	221	184	203
11	1.876	2.146	-----	1.118	.884	-----	372	346	362	-----	362	306	310
12	2.126	2.463	-----	1.706	1.323	-----	384	362	376	-----	376	326	363
13	2.093	2.398	-----	1.418	1.097	-----	546	510	532	-----	535	457	495
14A	2.022	2.447	-----	1.414	1.210	-----	492	460	476	-----	480	458	439
16	3.988	4.165	5.099	1.452	.772	1.658	112	114	112	109	105	45	78.9
16S	3.989	4.155	5.027	1.287	.732	1.703	113	116	114	110	106	36	87.2
17	1.591	1.596	6.429	.1095	.204	1.403	173	172	173	165	168	-----	200
18	3.824	3.920	4.466	.323	.173	2.561	218	219	218	218	203	240	189

[a]The calculated critical flutter speeds are corrected for compressibility.

[b]From tests in the LMAL 8-foot high-speed wind tunnel.

L-413

L-413

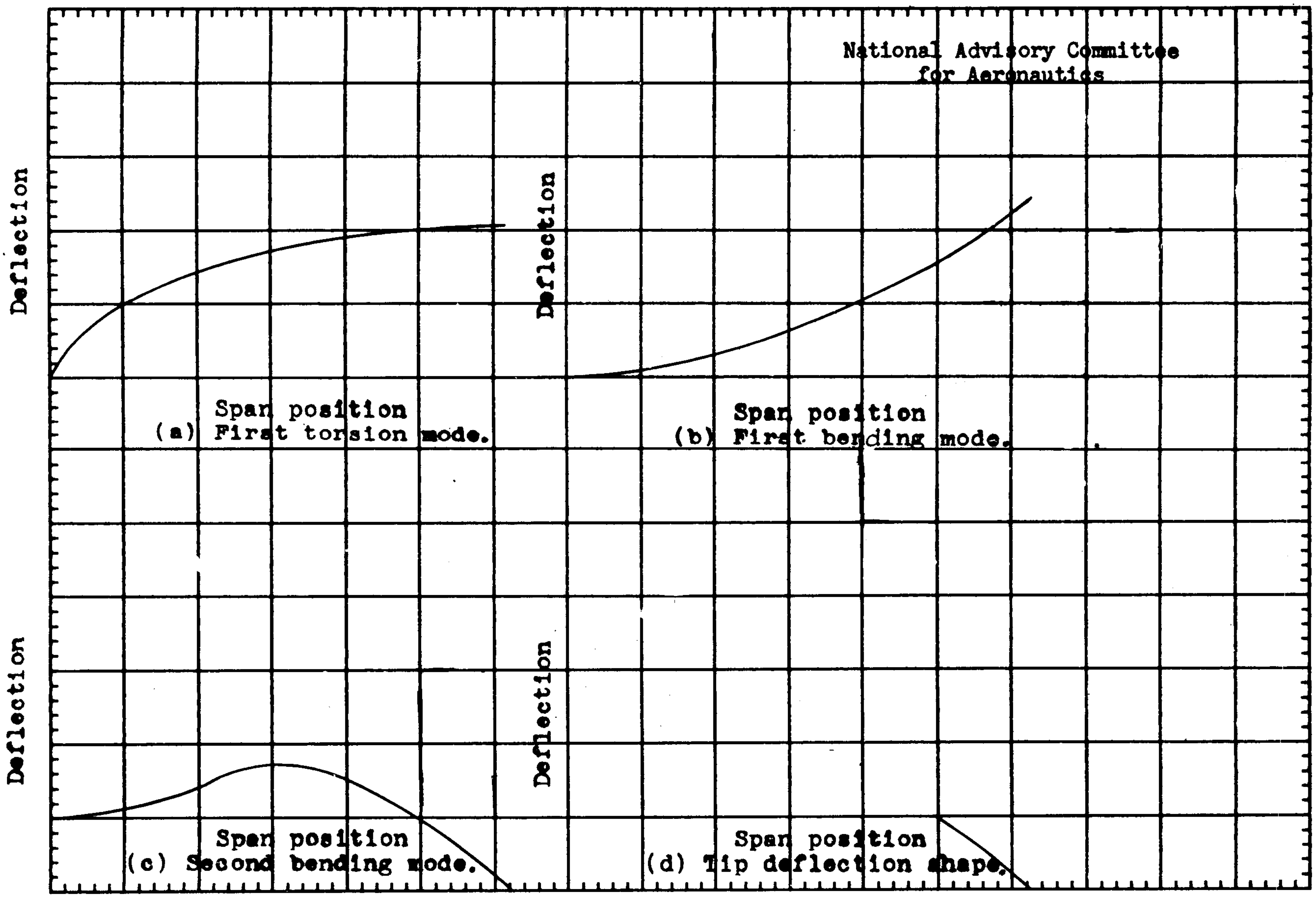

Figure 1. - Sketches illustrating the modes employed in the calculations.